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Novel Methods in Terminal Ballistics and Mechanochemistry of Damage: A Review of Developments at the US Army Research Laboratory, 2001–2007

by Michael Grinfeld

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14. ABSTRACT

This report is a review of the developments at the US Army Research Laboratory from 2001 to 2007 in the area of mechanochemistry of damage and its applications in terminal ballistics. It focuses on the radial cracking paradox in terminal ballistics. To justify the ballistic paradox we use two approaches: the sharp interface approach and the narrow damaged zone approach. The sharp interface approach is based on the concepts of tensorial chemical potentials and the stress-driven morphological instabilities. The narrow damaged zone approach is based on the Landau's order parameter concept in which the order parameter describes the fraction of the broken bonds.

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1. Introduction

This abbreviated report reviews the Phenomenological Mechanochemistry of Damage (PMD) theory and its potential contribution to terminal ballistics. It is not intended to provide a deep understanding of this subject. It just answers 2 questions, What is the PMD theory, and what are its basic techniques? As our Latin-speaking ancestors claimed, real understanding should answer not the relatively simple question, What is it? but rather the much more difficult questions, Why is it? and How did this come about? All these questions are addressed in the full review of PMD theory, which can be found on the Internet in Grinfeld.[1]

It is easier to read and understand the full review after mastering this report and getting a first, even superficial, impression of the PMD theory and its potential applications. Typically, the need for deeper understanding appears when the user faces real difficulties in applying suggested procedures. It is something that always happens with sufficiently deep fundamental research. Unavoidably, there are paradoxical inconsistencies in fundamentals. Those paradoxes are not fictitious—they are real, and they are much more reliable than the remaining elements of the theory.

The pragmatic success of fundamental research eventually results in novel techniques and instruments. However, they are often by-products of a preceding major failure and the efforts of overcoming the failure. For novices, facing a fundamental paradox is discouraging: I know this from my own experience when facing several paradoxical inconsistencies in the fundamentals of thermodynamics. Rigorous theory and engineering practice differ in many respects, and many frustrations in dealing with both sorts of problems seem unavoidable. But the fruitful attitude toward paradoxes should be positive, not negative. It is something that was clearly understood by practically all the giants like Einstein, von Neumann, Bohr, and Winer. Their inspiring writings helped me to overcome my methodological crisis and eventually resulted in developing the PMD for the needs of terminal ballistics. One hopes the opinions of these giants, partially reflected in the full review,[1] will also be helpful for the readers when and if they experience their own crisis dealing with the fundamentals of PMD.

The full review is intended to address several fundamental questions: What are the interrelations between the PMD on one hand and the Gibbs paradigm and its interpretation in Grinfeld[2] on the other?

One of Grinfeld's[2] 2 top results of is the discovery of the Stress Driven Rearrangement Instabilities (SDRIs) paradoxes. The SDRIs are established by means of the most reliable, rigorous thermodynamic theory. Yet the SDRI phenomena do not exist in nature. This discrepancy between the theory and reality is the essence of the SDRI paradoxes. From the standpoint of theoretical thermodynamics, their discovery is an achievement of

significance. From the standpoint of practical applications, it is nothing more than the major failure of the predictive ability of Gibbs' thermodynamical paradigm.[3] From a technical point of view, the PMD theory is the effort of adjusting the SDRI and the relevant mathematical tools to the needs of terminal ballistics. There are, then, questions of primary importance:

- How to reconcile the inexistence of SDRI phenomena in thermodynamics with the use of their analogies in the PMD theory?
- What is the history and motivation behind the suggested PMD theory?
- What is the key difference between the fundamental research and academic research?
- What is the role of art, religion, and aesthetics in fundamental and academic research?

Not all of these questions are addressed in the full review[1]; that will be done later. Why do we need the answers to these questions? It is because I strongly believe that, properly used, the PMD theory can bring a lot of useful results to academic and fundamental research by practioners and theorists. At the same time, I strongly believe that, without clear answers to those fundamental questions, the PMD theory can bring much more harm than good.

I still vividly remember a cartoon showing the hypothetic way in which Einstein allegedly discovered his famous formula $E = mc^2$. On a blackboard Einstein rejects the assumptions $E = ma^2$ and $E = mb^2$, putting a line through each. At last, with the exclamation "Eureka!", he finds the desired formula $E = mc^2$. Each of us is guilty of this sort of reconstruction of the histories of our predecessors' discoveries. In this way we made crucial mistakes, fooling ourselves and others. This is why we are not able to reach the level of our predecessors. Consequently, we keep living in the world of delusions, confusing our own numerous errors, superficial technical variations, and "improvements" with real progress.

In the beginning of my research career I laughed at this cartoon. Today, passing the zenith of my career, I know that these sorts of conjectures are not uncommon. Moreover, I myself am guilty of similar oversimplifications and misleading interpretations.

What do many of us do after initially acquiring a superficial understanding of the subject? We try to deepen our understanding by reconstructing the history of the subject. Our first reconstructions are extremely naive and superficial, as demonstrated in the cartoon. With such a legitimate but superficial interpretation of the history and the essence of relativity theory, the interpreter generates the "revolutionary" ideas of $E = md^2$ and then $E = me^2$. Where is the key mistake in this interpretation? It is in the fact that the real difficulty is not in suggesting the simple approach, but in rejecting hundreds of complex ones.

As we continue to study the topic, our reconstruction of the history changes essentially. The more realistic reconstruction opens the door for more productive ideas.

There is good news: Deepening historic reconstruction is not endless. Eventually, if one learns and understands the real history of his discipline, he or she will not find the ultimate self-consistent truth, as expected. Instead, he or she will surely find the real weaknesses of any fundamental theory under consideration, including mathematics, logic, thermodynamics, mechanics, relativity, quantum theory, etc. This is a universal fact of gnoseology; fundamentals of any human knowledge are self-contradictory and poorly understood.

The fundamental self-contradictions are widely known as paradoxes. Eliminating the fundamental contradictions is the real subject of fundamental research. In fact, eliminating contradictions in fundamentals is typically much easier than finding those contradictions. That is why we must know the history of appearance of the fundamental ideas. The purpose of this learning process is not in giving tribute to the greatest minds of the past. Rather, deep learning of the history and the driving motivations offers fundamental progress for today and the future. This is why I strongly recommend that readers whose interests go beyond immediate pragmatic needs eventually read the full review.[1]

2. Thermodynamic Roots of the PMD Theory

My own vision of the PMD has grown from studying the thermodynamics of heterogeneous systems.[2] The top achievements in the fundamental developments are always triggered by facing unexpected paradoxes. I define paradox as the major incompatibility of 2 major obvious facts.[4,5] When working with fundamentals of continuum theory of phase transformations of solids, I faced 2 major paradoxes. First was the paradox of multiple tensorial chemical potentials, which is in major contradistinction with the classical concept of the single scalar chemical potential. Second is a set of the paradoxes of the Stress Driven Morphological Instabilities (SDMIs).[6]

The PMD has been intensively studied at the US Army Research Laboratory (ARL) since 2001. Its applications in terminal ballistics include the novel theoretical concepts: tensorial chemical potentials, SDMIs, and Intensively Fractured Zones (IFZs), among others, which are just simplified versions of the SDRIs and Heterogeneous Systems (HSs) with sharp Phase Interfaces (PIs). Different tensorial chemical potentials have been introduced by Grinfeld[7,8]; other papers[9,10] were the first publications on the SDRIs. Numerous Grinfeld papers[7–12] contain paradoxical results of a different nature. They have been developed further in several dozen additional papers summarized in a monograph.[2] However, all of the remaining papers contain nothing paradoxical, and from the conceptual point of view should be treated as technical (i.e., logical) exercises of different levels of complexity and entertainment. It is important to realize from the beginning that PMD theory relies on the same mathematics as has Grinfeld.[2] At the same time, PMD theory deals with the totally different physics.

3. ARL Developments Related to PMD Theory

Progress in thermodynamics developed over many decades.[13] Of course, such a pace is inappropriate for engineering projects. At ARL I first tried to understand the vocabulary and mindsets of the US Army engineers. That was neither easy nor straightforward. If I managed to do so, it is due to the influence of von Neumann's writings.[14] In particular, I tried to follow his observation that

> the sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describe observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work—that is, correctly describe phenomena from a reasonably wide area. Furthermore, it must satisfy certain esthetic criteria—that is, in relation to how much it describes, it must be rather simple.[14]

One should avoid treating this quote as a primitive pragmatic vision of the scientific reality. Not all models deserve interest. Those that do must "satisfy certain esthetic criteria."[14]

It is important to emphasize that the approach described by von Neumann has very little in common with dealing with fundamentals of thermodynamics as I understand it.

I came to ARL in 2001 with the firm opinion that the SDRIs were just valuable theoretical paradoxes without any experimental grounds, much less any practical applications. Routine engineering modeling rarely needs advanced theoretical tools. The use of sophisticated models with rough experimental data is misguiding. Fortunately, I eventually discovered various photographs of bullets penetrating brittle plates similar to the illustration in Fig. 1, which shows the loss of axial symmetry by means of the appearance of radial cracks caused by the bullet penetrating through transparent armor (see many wonderful photographs in Strassburger et al.[15]). Those photographs stunned me because I noticed the loss of the original axial symmetry after axisymmetric bullets penetrated through apparently axisymmetric plate. The loss of symmetry is an easily noticeable paradox.

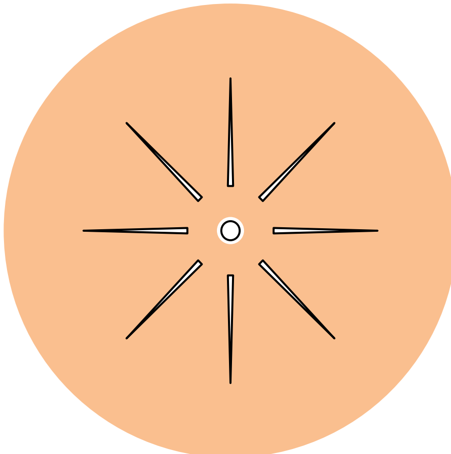

Fig. 1 The loss of axial symmetry by means of the appearance of radial cracks caused by bullet penetrating through transparent armor

What was the mechanism behind this paradox? I immediately (and almost subconsciously) noticed a striking similarity between those photographs and the computer-generated pictures obtained in modeling the SDRIs of PIs between solids phases (see Kassner et al.[18]). Because of my long-term experience in dealing with various instabilities, I immediately and subconsciously perceived a scenario where SDRI manifestation caused the loss of the axisymmetric solution and its replacement with the asymmetric one.

Contrary to previous modelers and experimenters, I noticed that the symmetry loss, combined with the radial cracking in the intact/comminuted transition, is the perfect setting for applying mathematics (not physics) to the SDRIs. Despite the potential of SDRI mathematics for terminal ballistics, the physics of the SDRIs offered just the opposite. What should I do with the obvious failure of the SDRI theory? How I can propose such a compromised theory? After months of hesitation, I chose the strategy based on the von Neumann's quote.

I decided to suggest the simple and, therefore, esthetically appealing theory that was expected to elegantly describe the patterns similar to those of Fig. 1. At the same time, I did not have the slightest intention to hide the truth about the fundamental failure of the SDRI theory from anyone, especially my coauthors. Quite the contrary: I always emphasized this difficulty to my colleagues. The elimination of this self-contradictory dichotomy should be the biggest driving force for further developments. One of the instruments of overcoming the dichotomy is the suggested PMD theory. It has, over time, resulted in obtaining (by means of combining the suggested theory with its computer implementation) the pattern of damage presented in Fig. 2, which shows the damage distribution for the case of the circular plate with a concentric hole and the axisymmetric loading.

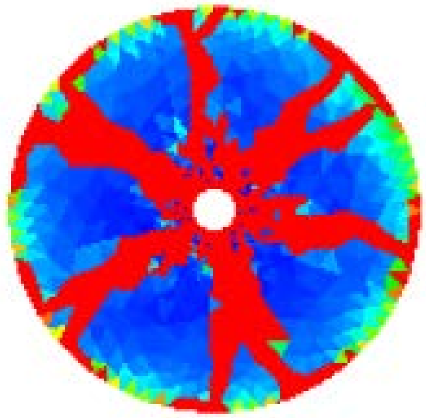

Fig. 2 Evolution of damage in a circular elastic plate with a concentric circular cavity under pressure

This pattern was found in 2005 by Pavel Grinfeld of Drexel University with the help of the mechanochemical model in MatLab (first published in 2007 in Grinfeld et al.[16]), which was one of the great breakthroughs in the development of PMD theory at ARL. The suggested PMD theory allowed us to cope with the ballistic paradox of symmetry loss. Basically, this is how the phenomenological mechanochemistry of damage came about. In the following, I present those developments with further technical details, which are much easier to understand than the conceptual difficulties and the history of appearance of the PMD theory.

4. Two Approaches to Modeling in the PMD Theory

For many decades, penetration of projectiles through various obstacles has remained the focus of many engineering disciplines, including military sciences. Many efforts have studied penetration into brittle materials.

A novel approach to these problems was cultivated at ARL beginning in 2000. It can be called the mechanochemical approach. There are 2 classes of models in mechanochemical approach: sharp interface models and continuously distributed damage models.

The term mechanochemistry is relevant for both classes of models. The sharp interface models widely use the concept of the tensorial chemical potentials as they were presented by Grinfeld.[2,7,8] The continuously distributed damage models rely on the concept of damage parameter, characterizing the fraction of broken chemical bonds in Kachanov.[17]

5. The Sharp Interface Model of the PMD

The sharp interface approach is widely used in terminal ballistics, for instance in the Johnson-Holmquist model. According to this model, 2 phases appear after the penetration of a solid projectile through a damageable substance: the comminuted phase and the intact phase. The key element in the sharp interface approach is the choice of the interface between the 2 phases.

Different criteria for the migration of the phase boundary can be suggested. The analogy between the traditional phases of Grinfeld[2] and the comminuted/intact phases prompted the relationship

$$J = -K\left[\mu^{ij}\right] n_i n_j \,, \tag{1}$$

where J is a mass flux through the interface, μ^{ij} is one of the tensorial chemical potentials, n_i is the unit normal to the phase interface, and K is a positive function, playing the role of the kinetic constant.

Kinetics condition 1 should be combined with 1) the bulk equation of momentum conservation

$$m\frac{\partial^2 u^i}{\partial t^2} = p^{ji}_{,\,j} \tag{2}$$

within each of the bulk domains, 2) the equation of mass conservation across failure/shock front

$$mc\left[\frac{\partial u^i}{\partial t}\right]_-^+ + \left[p^{ji}\right]_-^+ n_j = 0, \tag{3}$$

and 3) displacement u^i continuity condition across the failure front

$$[u^i]_-^+ = 0, \tag{4}$$

where m is the initial mass density of the substance, the derivative $p^{ji} = m\partial e\left(u_{p,q}\right)/\partial u_{i,\,j}$ defines the so-called Piola-Kirchhoff stress tensor, t = time, $a_{,\,j}$ defines differentiation with respect to the spatial coordinates x^j, and c and n_i are the velocity and unit normal to the front, respectively. The nonsymmetric tensorial chemical potential, associated with the elastic energy density $e\left(u_{p,q}\right)$, in the exact nonlinear form, reads

$$\mu_k^j = e\delta_k^j - m^{-1}p^{ji}(\delta_{ik} + u_{i,\,k}) \,. \tag{5}$$

The system described by Eqs. 1–5 does not contain any constraints on the smallness of the displacements or their gradients.

What makes kinetics Eq. 1 so appealing? It allows an immediate and natural description of the appearance of the radial cracks due to the so-called SDRIs of the phase interfaces, which were

both discovered analytically by Grinfeld[10,11] and eventually summarized by Grinfeld.[2] The appearance of cracking (rather than any other a priori possible patterns) as a result of the SDRIs of phase interfaces can be established only numerically. This work was done in Kassner et al.[18] and many other publications with the help of phase field numerical methods.

The key element of successful modeling is in the choice of the relevant elastic potentials for the intact and comminuted phases. The simplest possible model was suggested in Grinfeld and Wright,[19] shown schematically in Fig. 3. The elastic potential of the intact (comminuted) phase is shown in blue.

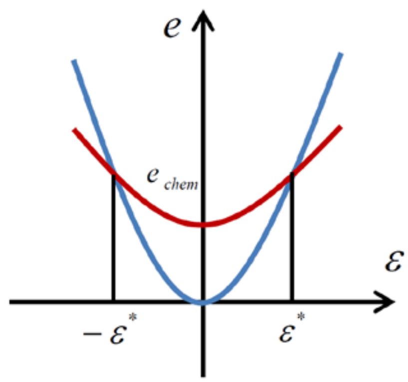

Fig. 3 Two-phase mechanochemical model of fracture

The simplicity of the germ model is of crucial importance. It should allow both analytical studies and reliable numerical implementation. In case of success with the simplest model, additional effects should be taken into account for a better match with experimental data. These developments unavoidably make the model less transparent conceptually and nontractable by analytical methods. However, it is worth sacrificing some elegance for the sake of making a useful tool for practitioners. This is much more reasonable than making generalizations for the sake of developing a universal theory.

The sharp interface approach for an intact/comminuted transformation, based on Eqs. 1–5, was suggested in Grinfeld and Wright.[19] It was further applied to the problems of the morphology of comminuted zones, failure waves, morphological stability of failure fronts, etc.[16,20–29]

When dealing with the SDRIs, it is crucial to keep in mind the important fact that the SDRIs do not exist in the case of traditional phase transformations in standard crystalline substances. Therefore, the results of the modeling, based on the suggested approach, should be carefully compared against experimental data. The very logical consistency of the theory, taken on its own, is neither necessary nor sufficient for practical applicability in terminal ballistics and other problems.

6. The Distributed Model of Mechanochemistry of Damage: The General Equations of Cold Mechanochemistry

In the distributed damage approach, there is no sharp boundary between the intact and comminuted phases. In fact, the elastic energy density e becomes the function not only of the displacement gradient $u_{i,j}$, but also of the damage parameter κ ($0 \le \kappa \le 1$):

$$e = e\left(u_{i,j}, \kappa\right) . \tag{6}$$

The following 2 equilibrium equations are associated with this model of substance. The standard equation of mechanical equilibrium reads

$$\frac{\partial}{\partial x^j} \frac{\partial e\left(u_{m|n}, \kappa\right)}{\partial u_{i|j}} = 0 . \tag{7}$$

The equilibrium with respect to damage/healing reads

$$\frac{\partial e\left(u_{m|n}, \kappa\right)}{\partial \kappa} = 0 . \tag{8}$$

In particular, for the uniform displacement gradients $u_{m|n}$ and damage parameter κ, Eq. 7 is satisfied, but the "chemical" equilibrium Eq. 8 is not satisfied automatically. The quantity $\chi \equiv \partial e\left(u_{m|n}, \kappa\right) / \partial \kappa$ is usually called the bulk chemical potential. The adjective bulk for the chemical potential χ was usually omitted. Traditionally, researchers did not distinguish between the bulk and the interfacial chemical potentials, instead using the chemical potential χ for both.[29] Only later did it become obvious that for logical consistency (not so much for the physical or engineering reasons) that the latter should be replaced with tensorial quantities.

For the distributed model of mechanochemistry of damage, the bulk master system reads

$$m \frac{\partial^2 u^i}{\partial t^2} = \frac{\partial}{\partial x^j} \frac{\partial e\left(u_{m|n}, \kappa\right)}{\partial u_{i|j}}, \tag{9}$$

and

$$\frac{\partial \kappa}{\partial t} = -K \frac{\partial e\left(u_{m|n}, \kappa\right)}{\partial \kappa}. \tag{10}$$

In the quasi-static case, dynamics Eq. 8 should be replaced with static Eq. 7.

The damage rate function K should be positive; it is the only thermodynamically motivated constraint imposed on this function. In fact, it can depend on the wide set of arguments

$\left(u_{m|n}, u_{m|np}, \partial u_{m|n}/\partial t, \kappa, \partial\kappa/\partial t, ...\right)$. At the same time, just the simplest assumption, $K = const > 0$, can be sufficient and instructive for various applications.

The case $\partial\kappa/\partial t > 0$ can be called active damaging, and the case $\partial\kappa/\partial t < 0$ can be called recovery. Actually, both regimes take place. In ballistics, though, the recovery is a much slower process than active damaging and often can be ignored. The simplest modification of Eq. 9 for such a case would the kinetics equation

$$\frac{\partial\kappa}{\partial t} = -\frac{\partial e\left(u_{m|n}, \kappa\right)}{\partial\kappa} \times \begin{Bmatrix} K_+ & \text{if} & \frac{\partial\kappa}{\partial t} \geq 0 \\ K_- & \text{if} & \frac{\partial\kappa}{\partial t} \leq 0 \end{Bmatrix}. \tag{11}$$

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7. The Mechanochemical Stability

The central idea of the PMD is to replace the nonphysical interface SDRIs with the Mechanochemical Bulk Instabilities (MBIs). Generally speaking, the occurrence of MBIs depends on various details relating to the external loading. It makes sense, though, to extract from all the different manifestations of the MBIs those that do not depend upon the external loading conditions.

The concept of the mechanochemical stability of solids can be illustrated by comparison with the problem of thermodynamic inequalities in the classical theory of elasticity. In general, the stability conditions of any elastic structure obviously depend on the structure's geometry and boundary loads. However, much more general thermodynamic inequalities include no information about those circumstances. In this sense, they are equally related to all the structures made of a given material. Thermodynamic inequalities, then, characterize the material and not structures made of this material. Fulfilling thermodynamic inequalities are the mandatory conditions for stability of any structure. Those mandatory conditions are necessary but far from being the sufficient conditions of stability conditions of the structure.

The mechanochemical stability conditions for the PMD theory have been suggested by Grinfeld and Grinfeld.[28] For the local stability of the equilibrium configuration $\left(u^{\circ}_{m|n}, \kappa^{\circ}\right)$, the following inequalities must be satisfied[28]:

$$e^{ijkl\circ} l_i l_k n_j n_l \geq 0,\ \left(e_{\kappa\kappa} e^{ijkl\circ} - e_{\kappa}^{ij\circ} e_{\kappa}^{kl\circ}\right) l_i l_k n_j n_l \geq 0,\ e_{\kappa\kappa} \geq 0, \tag{12}$$

for any two real vectors l_i, n_j. These inequalities are the generalized Legendre necessary condition for the minimum of the integral functional with the integrand $e\left(u_{m|n}, \kappa\right)$ (compare with the thermodynamic inequalities as they interpreted in Grinfeld[2]). The first of the mechanochemical stability conditions, 12 – $e^{ijkl\circ} l_i l_k n_j n_l \geq 0$, can be called the hyperbolicity

condition of stability. The last one, $e_{\kappa\kappa} \geq 0$, can be called the chemical condition of stability. The remaining condition, $\left(e_{\kappa\kappa} e^{ijkl\circ} - e_{\kappa}^{ij\circ} e_{\kappa}^{kl\circ}\right) l_i l_k n_j n_l \geq 0$, can be called the mechanochemical condition of stability (in the narrow sense.)

8. The Engineering (Elastically Linear Isotropic) Model

Two ingredients of the internal energy of the target were taken into account. The first is the traditional elastic energy, and the second is the energy associated with the chemical bonds. In addition to the standard independent variable of elasticity theory, one more variable, the damage parameter, is taken into account. In our model, the damage parameter influences both the elastic and chemical ingredients of the internal energy.

In the simplest case, the relevant function $e\left(u_{i,j}, \kappa\right)$ at fixed κ should look like those shown in Fig. 3. At $\kappa = 0$ we get the intact phase, and at $\kappa = 1$ we get the fully comminuted phase.

A reasonable first choice could be to choose the mechanochemical potential in the following form:

$$e_{\text{int}}\left(\varepsilon_{ij}, \kappa\right) = \varphi\left(\kappa\right) \mu \left(\frac{\nu}{1-2\nu} \left(\varepsilon_i^i\right)^2 + \varepsilon_{ij} \varepsilon^{ij} \right) + \frac{\xi}{2} \kappa^2, \tag{13}$$

with the damage function $\varphi(\kappa)$ as follows:

$$\varphi\left(\kappa\right) = 1 - \left(1 - c_{\min}\right) \frac{\kappa}{\kappa^*}, \tag{14}$$

where

$$0 \leq \kappa \leq \kappa^*,\ 0 < c_{\min} \leq 1, \tag{15}$$

as was suggested by Grinfeld and Wright.[19,20]

Thus, the suggested model depends on 5 constants: μ, ν, ξ, κ^*, and $c_{\min}$. The first 2 are just the shear modulus and Poisson's ratio of the intact substance; the physical meaning of the remaining 4 constants is explained elsewhere.

It makes sense to consider a close model,

$$\varphi\left(\kappa\right) = 1 - \alpha\kappa, \tag{16}$$

with a positive constant α, and the damage parameter belonging to the range $0 \leq \kappa \leq 1$. The constant α can be either greater or smaller than 1. If it is smaller than 1, the current elastic modules vanish before the solid completely loses its integrity at $\kappa = 1$. This leads us to the

analysis of the dynamic states with negative current elastic modules. Such states cannot occur in the stable equilibrium configurations. However, theoretically they can show up in transient configurations.

9. Computer Implementation of the PMD: The MatLab Implementation

The first computer implementation of the PMD system of Eqs. 9–11 was accomplished by Pavel Grinfeld of Drexel University.[16,21] The PMD was implemented using the finite element method in MatLab for the quasi-static approximation (i.e., when the inertia term in the bulk Eq. 9 is neglected). Figure 4 shows the damage evolution around an elliptic hole in a circular plate pressurized at the outer boundary.[16,21] We clearly see the process of fingering of the intensively damaged zones in the vicinity of the ellipse's tips.

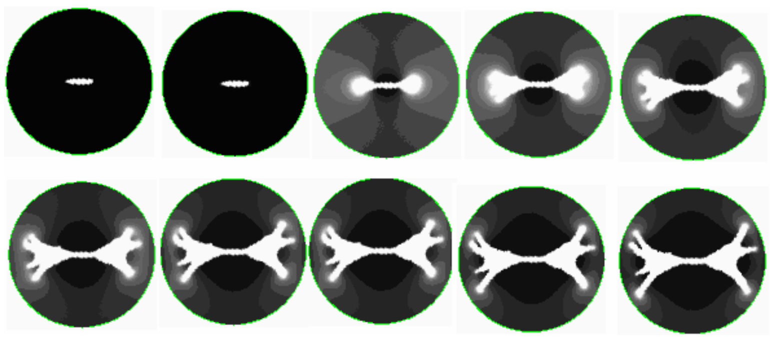

Fig. 4 Evolution of damage in a circular elastic plate with an elliptic cavity

The crucial question for the PMD model is the possibility of explaining the paradox of radial cracking in the radially symmetric problem with the circular hole. Without the scenario of radial cracking, the PMD model would be worthless, and the existence of this scenario was not at all guaranteed. The ultimate goal of the project was a computer-based analysis of the possibility of radial cracking. Fortunately, radial cracks for the quasi-static distributed model of mechanochemistry, described by Eqs. 9–12, indeed did show up. Pavel Grinfeld's pattern is shown in Fig. 2.

The patterns shown in the Figs. 2 and 4 are of immense importance for the PMD theory and its applications in terminal ballistics (among other engineering and scientific disciplines).

10. Conclusions

In summary, the origin and early developments of the PMD theory at ARL were triggered by an attempt to explain the radial cracking paradox, often observed in terminal ballistics of brittle armor. This pattern recalls the patterns appearing at the late stage of the SDRIs in the thermodynamics of phase interfaces separating different solid phases. However, the SDRIs of phase interfaces are thermodynamical paradoxes rather than real physical phenomena. Therefore, there appeared to be the necessity to suggest a novel physical interpretation of the mathematical construct leading to the desired pattern of the radial cracking. The PMD theory was suggested as a response to this necessity.

The PMD theory has explained the radial cracking paradox with remarkable ease, as demonstrated by Fig. 2. The PMD model is so simple that it permits both computer-based and analytical study. This success inspires hope for making further progress in describing the real pattern shown in Fig. 2. However, we are now very far from that level of success.

In no way can the suggested model be treated as the ultimate truth from the standpoint of physics. We know for sure that the original SDRI theory, which is a prototype of PMD, has failed in much more favorable situations in the theory of phase transformations.

The current simplicity of the PMD theory makes it not only promising, but also fragile. One of the biggest risks for the PMD theory is to degenerate into purely formal exercises and not rely on the robust experimental data. This happened recently with the SDRI phenomena. John von Neumann compared this risk with degenaration of the classical style into baroque style in architecture.[14]

Other approaches that allow modeling of radial cracking are developed and reviewed in Grady,[31] Brannon et al.,[32] and Leavy et al.[33] As with any human theories, their approaches have both pros and cons. Of course, human theories compete; this is a long-term evolutionary process that requires several generations of researchers. This long-term process has nothing in common with the short-term process of competition between individual researchers. This competition is not about trial and error, logical mistakes, and eloquent speeches, academic paper writing, and practical needs. It is all about esthetics and psychology.

Which of the approaches will eventually prevail? This is a very interesting question, one that was addressed by von Neumann, a long-term collaborator with several government organizations including the US Army Ballistic Research Laboratory at Aberdeen Proving Ground, MD. In 1947 he wrote, discussing the competition between theories,

> The decision (in favor of one of the competing theories...M.G.) is likely to be opportunistic in the end. The theory that lends itself better to formalistic extension

> towards valid new theories will overcome the other, no matter what our preference up to that point might have been. It must be emphasized that this is not a question of accepting the correct theory or rejecting the false one. It is a matter of accepting that theory which shows greater formal adaptability for a correct extension. This is formalistic, aesthetic criterion, with a highly opportunistic flavor.[14]

The suggested PMD theory is now quite elementary. Even undergraduates with modest mathematical skills can handle it. But the distances between formal handling, real understanding, and the ability to suggest a novel simple theory are great. The real difficulty is not in handling the mathematics, but in the ability to reject hundreds of more complex approaches. I hope this introduction will be the first tiny step in covering these distances.

The PMD theory should compete mostly with experimental data and unnecessary complexity. I believe it has great potential for such a healthy competition. Theories die not because they are wrong in one respect or another—every theory is wrong in this sense. In fact, theories die only due to their imbalanced overcomplexity entailing the loss of aesthetical appeal. Aesthetics, though, include several ingredients; this important discussion will be presented elsewhere.

11. References and Notes

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3. In fact, the same can be said about the first and second laws of thermodynamics: the impossibility of perpetuum mobile is a significant achievement for thermodynamics. However, from practical point of view it is a major failure for designers of the perpetuum mobiles. There is an essential difference, though, between the usage of the first and second laws of thermodynamics and the use of the SDRIs in the suggested applications of the PMD in terminal ballistics. Practically no one tries anymore to design the perpetuum mobiles, prohibited by the first and second laws of thermodynamics. At the same time, we deliberately propose to use the already failed SDRI theory in terminal ballistics.

4. Paraphrasing Niels Bohr,[5] There are trivial truths and great truths. The opposite of a trivial truth is plainly false. The opposite of a great truth is also true.

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13. For instance, the second energy variation for the system "crystal-melt" was found in 1982, more than a century after Gibbs calculated the first variation. Several powerful mathematical tools should be developed to make that happen. Even more importantly, it was necessary to realize that the classical textbook approaches to the Gibbs paradigm were unacceptable.

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